Exam Radiative Processes

(April 6, 2011 – 13.00-16.00, Room ZG257)

The number between brackets indicates the maximum score for that question.
Please indicate your name and student number on each sheet that is handed in.
Also think about using the correct units.

Conceptual Questions:

1. Make use of Maxwell's equations and the concept of field-lines and retarded time to explain, in words and maybe a drawing, why accelerating charges emit radiation and non-accelerating charges do not. Think about the electric field-line structure of a charge either at rest, moving with a constant velocity, or accelerating. [10]

2. (a) Write down the equation for the total power (integrated over solid angle) emitted by a non-relativistic electron moving in a constant magnetic field. What is this radiation called? What is this radiation called if the source *is* moving relativistically. Why in the former case is the radiation seen by an observer nearly narrow band (i.e. spread over a narrow frequency range) and in the latter case wide-band? [10]

   (b) Explain in words how one can interpret this equation and where the emitted power comes, using the point of view of what the moving electron "sees" while moving through the magnetic field. [5]

   (c) Now also write down the equation for the power emitted by a *relativistic* electron moving in a magnetic field. Explain where the terms $\gamma = 1/\sqrt{1-\beta^2}$ and $\beta = v/c$ in this equation come from in terms of Lorentz transformations from the electron rest-frame to that of the observer? Use drawings if necessary. [Hint: Think of energy and geometry.] [10]

   (d) If you think of photons as a time-varying electro-magnetic field, write down the photon energy density in terms of the photon magnetic field strength (assuming for simplicity monochromatic radiation) and explain how, conceptually, inverse Compton radiation is related the radiation discussed in (a)-(c). [5]

Calculation Questions:

3. A radio source with flux density $S_\nu = 1\,\mathrm{Jy} = 10^{-26}\,\mathrm{W\,m^{-2}\,Hz^{-1}}$ is measured with a radio-telescope to have an angular diameter of 5 milliarcseconds at 1 GHz. What is the brightness temperature of this source? [5]

4. Show that if stimulated emission is neglected, leaving only two Einstein coefficients, an appropriate relation between the coefficients will be consistent with thermal equilibrium between the atom and a radiation field of a Wien spectrum, but not of a Planck spectrum. [15]

5. The interstellar medium of our Galaxy contains dust particles [assume they have a radius a] and starlight (assume $\mathrm{T}_c = 10^4\mathrm{K}$) that bathes the dust particles in dilute radiation field. Assume that the fraction of sky "covered" with stellar photospheres is $w = 10^{-14}$, so that $J_\nu \approx w B_\nu(T_c)$.

   (a) Estimate the equilibrium temperature of the dust particles. [5]

   (b) At what wavelength would a blackbody emitting at this temperature have the largest intensity? [5]

6. Show that the Larmor radius of a proton, moving at 10 km sec^{-1} through a field of 10^{-10} Tesla, is small compared to interstellar and even interplanetary distances. [Hint: Make use of the Larmor frequency $\omega_{\mathrm{L}} = eB_{\perp}/(m_{\mathrm{e}})$.] [10]

7. An ultrarelativistic electron emits synchrotron radiation.

   (a) Show that its energy decreases with time according to [10]

$$\gamma = \gamma_0(1 + A\gamma_0 t)^{-1}$$

   where A is a constant (note that you do not need to derive the constant) and γ_0 is the initial value of γ.

   (b) What is the time for the electron to lose half of its energy? [10]

Good luck!

Constants (SI):
Electron charge $e = 1.602177 \cdot 10^{-19}\,\mathrm{C}$
Electron mass $m_{\mathrm{e}} = 9.109389 \cdot 10^{-31}\,\mathrm{kg}$
Boltzmann's Constant $k = 1.3806 \cdot 10^{-23}\ \mathrm{J\ K^{-1}}$
Planck's Constant $h = 6.64 \cdot 10^{-34}\ \mathrm{J\ s}$
Stefan-Boltzmann Constant $\sigma_{\mathrm{SB}} = 5.6703 \cdot 10^{-8}\ \mathrm{J\ s^{-1}\ m^{-2}\ K^{-4}}$
Speed of Light $c = 2.99792458 \cdot 10^{8}\ \mathrm{m\ s^{-1}}$